

The deformation of the interacting nucleon in the Skyrme model

A. Rahimov

Theoretical Physics Department, Tashkent State University
700095 Tashkent, Uzbekistan

T. Okazaki

Physics Laboratory, Sapporo Campus, Hokkaido University of Education,
Sapporo, Japan

M.M. Musakhanov*, F. C. Khanna[†]

Department of Physics, University of Alberta
Edmonton, Canada T6G 2J1

and

TRIUMF, 4004 Wesbrook Mall,
Vancouver, British Columbia,
Canada, V6T 2A3,

February 9, 2008

Abstract

Changes in the nucleon shape are investigated by letting the nucleon deform under the strong interactions with another nucleon. The parameters of the axial deformations are obtained by minimizing the static energy of the two nucleon system at each internucleon distance R . It is shown that the intrinsic quadrupole moment of the interacting proton, Q_p , is about $0.02 fm^2$ at distances near $R \sim 1.25$ fm.

[PACS numbers: 13.75 Cs , 11.10 Lm;
nucleon nucleon interaction, nucleon shape, quadrupole moment]

*E-mail: yousuf@phys.ualberta.ca; permanent address: Theor.Phys.Dept., Tashkent State University

[†]E-mail: khanna@phys.ualberta.ca

The possible changes in the radius of a nucleon in interaction with another nucleon was investigated in the skyrmion model by Kalbermann et. al. [1]. It was found that there is a swelling of the nucleon at intermediate distances between the nucleons which were assumed to preserve their spherical shape. On the other hand C. Hajduk and B. Schwesinger [2] considering the skyrmion to be soft for the deformation showed that there are several deformed states of rotating skyrmions. In particular, besides the ground state of a spherical shape there exists a degenerate doublet of exotic states with the same quantum numbers as the nucleon ($s = t = 1/2$) of oblate and prolate shapes. One may therefore wonder whether the nucleon can change its shape under the action of strong interactions.

Certainly, several studies have been made of the deformation effects on the Skyrmions [3], [4], [5], but these were carried out to obtain a better understanding of the possible sources of attraction in the central nucleon - nucleon potential. As a result it was shown that the deformation effect is very important and may reduce the central repulsion by about 40%.

In a previous paper [6], the problem of the missing central attraction in the NN - interaction has been investigated within the model of Andrianov and Novozhilov [7], [8] which starts with the Skyrme lagrangian supplemented by a dilaton scalar field $\sigma(r)$ so as to satisfy the QCD trace anomaly constraint. It was concluded that this model gives the desired attraction in the central part of the NN interaction. The resulting central scalar-isoscalar part of the potential is in qualitative agreement with the phenomenological one.

In the present paper we shall concentrate on the effects of the modification of the shape of a nucleon at a quantitative level using skyrmions. We shall use the same model [7], [8]. The lagrangian of this model including the chiral symmetry breaking term $\mathcal{L}_{\chi\text{sb}}$ has the form:

$$\begin{aligned}
\mathcal{L}(U, \sigma) &= \mathcal{L}_2(U, \sigma) + \mathcal{L}_{4a}(U) + \mathcal{L}_w(\sigma) + \mathcal{L}_{\chi\text{sb}}(U, \sigma), \\
\mathcal{L}_2(U, \sigma) &= -\frac{F_\pi^2}{16} [\text{Tr } L_\mu L^\mu - 2(\partial_\mu \sigma)^2] e^{-2\sigma}, \\
\mathcal{L}_{4a}(U) &= \frac{1}{32e^2} \text{Tr } [L_\mu, L^\nu]^2, \\
\mathcal{L}_w(\sigma) &= -\frac{C_g}{24} [e^{-4\sigma} - 1 + \frac{4}{\varepsilon}(1 - e^{-\varepsilon\sigma})] \\
\mathcal{L}_{\chi\text{sb}}(U, \sigma) &= \frac{e^{-3\sigma} m_\pi^2 F_\pi^2}{8} \text{Tr } (U - 1)
\end{aligned} \tag{1}$$

where F_π is the pion decay constant, m_π is mass of the pion, e is the coupling constant of the Skyrme term \mathcal{L}_{4a} , C_g is gluon condensate parameter σ is scalar meson field, U is an $SU(2)$ matrix chiral field and $L_\mu = U^\dagger \partial_\mu U$. It is a generalization of the well - known original Skyrme model [9], [10] and takes into account the conformal anomaly of the QCD. The term \mathcal{L}_2 includes the kinetic term of the chiral and scalar fields. The effective potential for the scalar field was calculated by Migdal and Shifman [11]. The parameter ε depends on the number of flavors N_f : $\varepsilon = 8N_f/(33 - 2N_f)$. Note that in the limit of a heavy σ -meson the potential becomes equal to the symmetric quartic term $\mathcal{L}_{4s} = \frac{\gamma}{8e^2} [\text{Tr } L_\mu L^\mu]^2$, which is necessary to reproduce the $\pi\pi$ scattering data [12]. The main difference between the model of refs. [13], [1], [14], [16] and this one (eq.(1)) is in the origin of the dilaton. In the former [13] the dilaton is associated with the glueball while in the latter [7] it is associated with the quarkonium. Nevertheless both models produce in a natural and transparent way the intermediate - range attraction in the central part of baryon - baryon interaction even in the product approximation [14]. We assume that both skyrmions deform into an ellipsoidal shape as they come close together. To describe this we write the chiral field U and the dilaton field σ as the nonspherical hedgehog form given by :

$$\begin{aligned} U_0(\vec{r}) &= \exp(i\vec{\tau}\hat{q}\Theta(q)) \\ \sigma(\vec{r}) &= \sigma(q) \end{aligned} \quad (2)$$

where the spatial vector \vec{q} has the components $\delta_x x, \delta_x y, \delta_z z$, \hat{q} is the unit vector: $\hat{q} = \vec{q}/q$ with the deformation parameters δ_x and δ_z . The profile functions Θ and σ are assumed to be the solutions of the Euler - Lagrange equations in the spherical case given by [4]. This ansatz, eq. (2), leads to a modification of the static mass, M_H^* , and the moment of inertia, λ_M^* , of the Skyrmion

$$\begin{aligned} M_H^* &= [AM_2 + BM_{4a} + M_{\chi sb} + M_W]/\eta \\ \lambda_M^* &= [\lambda_2 + A\lambda_{4a}]/\eta \end{aligned} \quad (3)$$

where $\eta = \delta_x^2 \delta_z$, $A = (2\delta_x^2 + \delta_z^2)/3$, $B = \delta_x^2(\delta_x^2 + 2\delta_z^2)/3$ and M_i and λ_i denote the relevant contributions from \mathcal{L}_i term in eq. (1) for the spherical case $\delta_x = \delta_z = 1$. As we are mainly interested in the region where the medium range attraction takes place - $R \sim 1.25 fm$ (typical separation between nucleons in nuclei) we restrict ourselves to the familiar product ansatz:

$$\begin{aligned} U &= \hat{A}_1 U_0(\vec{X} - \vec{q}/2) \hat{A}_1^\dagger \hat{A}_2 U_0(\vec{X} + \vec{q}/2) \hat{A}_2^\dagger \equiv U_1 U_2 \\ \rho &= \rho(\vec{X} - \vec{q}/2) \rho(\vec{X} + \vec{q}/2) \equiv \rho_1 \rho_2, \quad \rho \equiv \exp(-\sigma) \end{aligned} \quad (4)$$

where \hat{A}_1, \hat{A}_2 are the collective coordinates of skyrmions to describe their rotational motion, \vec{q} is the vector along z axis: $q_x = 0, q_y = 0, q_z = q = R\delta_z$ and R is the distance between skyrmions. The static skyrmion - skyrmion potential is defined by:

$$V(\vec{R}, \delta_x, \delta_z) = -\int d\vec{X} [\mathcal{L}(U_1 U_2, \rho_1 \rho_2) - \mathcal{L}(U_1, \rho_1) - \mathcal{L}(U_2, \rho_2)] \quad (5)$$

The application of the usual projection methods developed in [17], [18], [19], [20], [21] to eq.s (1) – (5) yields the following representation for the central scalar - isoscalar part of the nucleon - nucleon interaction :

$$V_c(R, \delta_x, \delta_z) = \frac{1}{\eta} [V_{\chi sb}(q) + V_W(q) + \delta_x^2 (V_2(q) + \delta_z^2 V_{4a}(q)) + (\delta_z^2 - \delta_x^2) (V_2^{def}(q) + \delta_x^2 V_{4a}^{def}(q))] \quad (6)$$

where the terms V_2^{def} and V_{4a}^{def} are the net contributions from the deformation effect of the terms \mathcal{L}_2 and \mathcal{L}_{4a} in eq.(1) respectively. The deformation parameters $\delta_x(r)$ and $\delta_z(r)$ were calculated by minimizing the total static energy of the two nucleon system at each separation R using eq. (2) and eq. (6).

The resulting values of the parameters are then used to study the changes in the shape of the nucleon. We will illustrate this procedure for the case of the isoscalar mean square radius and the appropriate intrinsic quadrupole moment. The normalized isoscalar mean square radius along each axis may be defined by:

$$\langle r_i^2 \rangle_{I=0}^* = \frac{\int d\vec{r} r_i^2 B_0(\vec{r})}{\int d\vec{r} B_0(\vec{r})} \quad (7)$$

where ($i = x, y, z$), $B_0(\vec{r})$ is the baryon charge distribution

$$B_0(\vec{r}) = \frac{1}{24\pi^2} \epsilon^{ijk} \text{Tr} [L_i L_j L_k]. \quad (8)$$

The inclusion of the deformation in a simple way : $r_i \rightarrow q_i / \delta_i$ as in eq. (2) yields the following relation between the radius of a free spherical nucleon $\langle r^2 \rangle_{I=0}$ and a deformed one : $\langle r_i^2 \rangle_{I=0}^* = \frac{1}{\delta_i^2} \langle r^2 \rangle_{I=0}$. Therefore the appropriate quadrupole moment characterizing the shape of the baryon matter distribution is compared to that of an ellipsoid with axis $1/\delta_z$ and $1/\delta_x$: $Q_{I=0} = 3\langle r_z^2 \rangle_{I=0}^* - \langle r^2 \rangle_{I=0}^* = 2\langle r^2 \rangle_{I=0} (1/\delta_z^2 - 1/\delta_x^2)$. The explicit formulas for $Q_{I=1}$ defined by $Q_{I=1} = 3\langle r_z^2 \rangle_{I=1}^* - \langle r^2 \rangle_{I=1}^*$ are rather complicated and may be found elsewhere [22].

In the numerical calculations we consider the following two cases: the lagrangian with the dilaton and the pure Skyrme model when $\sigma = 0$ in eq. (1). In both cases the parameters F_π , e and m_π were fixed at the values: $F_\pi = 186MeV$, $e = 2\pi$, $m_\pi = 139MeV$. For the gluon condensate we use $C_g = (283MeV)^4$ as obtained from lattice QCD calculations [23]. The mass of the scalar meson, m_σ , defined by $m_\sigma = \sqrt{2C_g}/F_\pi$ is then $610MeV$.

This set of parameters produce the following static properties of the nucleon: $M_N = 1054MeV$, $g_A = 0.65$, $\langle r^2 \rangle_{I=0}^{1/2} = 0.38fm$ and $\langle r^2 \rangle_{I=1}^{1/2} = 0.66fm$ in the dilaton case. No attempt is made here to search for a realistic set of parameters since our interest is mainly to establish a link between the properties of NN interaction and the shape of the nucleon.

In Fig.1 the central scalar - isoscalar part of the nucleon - nucleon interaction has been presented for the two cases (both including deformation effects) with the dilaton field (solid curve) and without one (dashed curve). For comparison the realistic phenomenological "Paris" potential [24] is also displayed (the dotted line in Fig.1). It is clear that the lagrangian with the dilaton field is able to describe the nucleon - nucleon interaction in the intermediate region quite well.

In the case with the dilaton field, the deformation effects give a contribution to the the central scalar - isoscalar part of the nucleon - nucleon potential $\sim -2MeV$ at the minimum point $R \sim 1.25fm$. It means that an attraction in the central part of NN interactions is provided mainly by the dilaton field.

In addition to the static (adiabatic) potential, there are also dynamical R dependent effects in the reduced mass of NN system. This dependence gives rise to a velocity dependent attraction. However, it is gives a small contribution at low energies [15].

We now turn to changes in the shapes of the interacting nucleons. The intrinsic quadrupole moments $Q_{I=0}$ and $Q_{I=1}$ are shown in Fig.2 and Fig.3 respectively. In the pure Skyrme model when there is no attraction ($V_2 = V_2^{def} = 0$ in eq. (6)) between skyrmions it becomes oblate (dashed lines in the Figures 2, 3) due to the strong repulsion caused by the V^4 terms in eq. (6). The inclusion of the dilaton leads to the following qualitative picture: At large separations skyrmion is obviously in a spherical shape, becomes prolate at the intermediate region and deforms to an oblate shape at small distances where the repulsion dominates. As the nucleons approach each other they change shapes from prolate into oblate at $R \sim 1.2fm$. Comparing Figures 2 and 3 it may be noticed that the isoscalar

intrinsic quadrupole moment $Q_{I=0}$ is much smaller than the isovector one $Q_{I=1}$ at intermediate separations.

The intrinsic quadrupole moment of the proton defined by: $Q_p = (Q_{I=0} + Q_{I=1})/2$ reaches a maximum value of $Q_p = 0.016 fm^2$ at $r \sim 1.5 fm$. Hence, one may conclude that the shape of a nucleon in nuclei is not spherical. We expect new data from high - energy electron scattering on nuclei to make this situation clear.

As a concluding remark we have to underline that the deformed states of oblate (prolate) shapes may not necessarily belong to the $K = 1$ band [2] since for a strongly deformed system the quantization procedure used here needs some modifications.

This work was supported in part by International Science Foundation Grant MZJ000 and JSPS grants. The work of F.Khanna is supported in part by the Natural Sciences and Engineering research Council of Canada.

References

- [1] G. Kalbermann, L. L. Frankfurt and J. M. Eisenberg, *Phys. Lett.* **B329** (1994) 164.
- [2] C. Hajduk and B. Schwesinger, *Nucl. Phys.* **A453** (1986) 620.
- [3] M. Oka, *Phys. Rev.* **C36** (1987) 720.
- [4] S. Saito, *Progr. Theor. Phys. (Supplement)* **91** (1987) 181.
- [5] U. B. Kaulfuss and U. -G. Meissner, *Phys. Rev.* **D31** (1985) 3024.
- [6] M.M. Musakhanov and A.M. Rahimov, *Mod. Phys. Lett.* **A10** (1995) 2297.
- [7] V. Andrianov and V. Novozhilov, *Phys. Lett.* **B202** (1988) 580.
- [8] V. A. Nikolaev, V. Novozhilov and O. G. Tkachev, *Preprint of JINR* (Rapid communications) **2** [53] - **92** (1992).
- [9] T. H. R. Skyrme, *Nucl. Phys.* **31** (1962) 556.
- [10] G. S. Adkins, C. R. Nappi and E. Witten, *Nucl. Phys.* **B228** (1983) 552.
- [11] A. Migdal and M. Shifman, *Phys. Lett.* **B114** (1982) 445.

- [12] J. Donoghue, E. Golowich and B. Holstein, *Phys. Rev. Lett.* **53** (1984) 747.
- [13] H. Gomm, P. Jain, R. Johnson and J. Schechter, *Phys. Rev.* **D33** (1986) 819;
ibid 3476.
- [14] H. Yabu, B. Schwesinger and G. Holzwarth, *Phys. Lett.* **B224** (1989) 25.
- [15] M. Oka, *Nucl. Phys.* **A463** (1987) 247c.
- [16] T. Wainzoch and J. Wambach, *Phys. Lett.* **B295** (1992) 16.
- [17] A. Jackson, A. D. Jackson and V. Pasquer, *Nucl. Phys.* **A432** (1985) 567.
- [18] T. Otofujii et al., *Phys. Rev.* **C34** (1986) 1559.
- [19] A. De Pace, H. Mütter and A. Faessler, *Z. Phys.* **A325** (1986) 229.
- [20] E. M. Nyman and D. O. Riska, *Phys. Scripta* **34** (1986) 533.
- [21] H. Yabu and K. Ando, *Progr. Theor. Phys.* **74** (1985) 750.
- [22] M. M. Musakhanov and A. Rahimov, submitted to *Yad. Phys.* (1995).
- [23] H. Satz, *Phys. Lett.* **B113** (1982) 245.
- [24] M. Lacombe et al., *Phys. Rev.* **C21** (1980) 861.

Figure captions

Fig.1. Central isospin - independent potential (eq. 6) calculated for the cases with the dilaton field (solid curve) and without one (dashed curve) as a function of the internucleon distance R . The dotted curve is the corresponding interaction component in the "Paris" potential [24].

Fig.2. The R dependence of the isoscalar quadrupole moment $Q_{I=0}$ of the nucleon. The solid and dashed lines are obtained in the case with dilaton and pure Skyrme model respectively.

Fig.3. The same as in Fig.2 but for the isovector quadrupole moment $Q_{I=1}$.

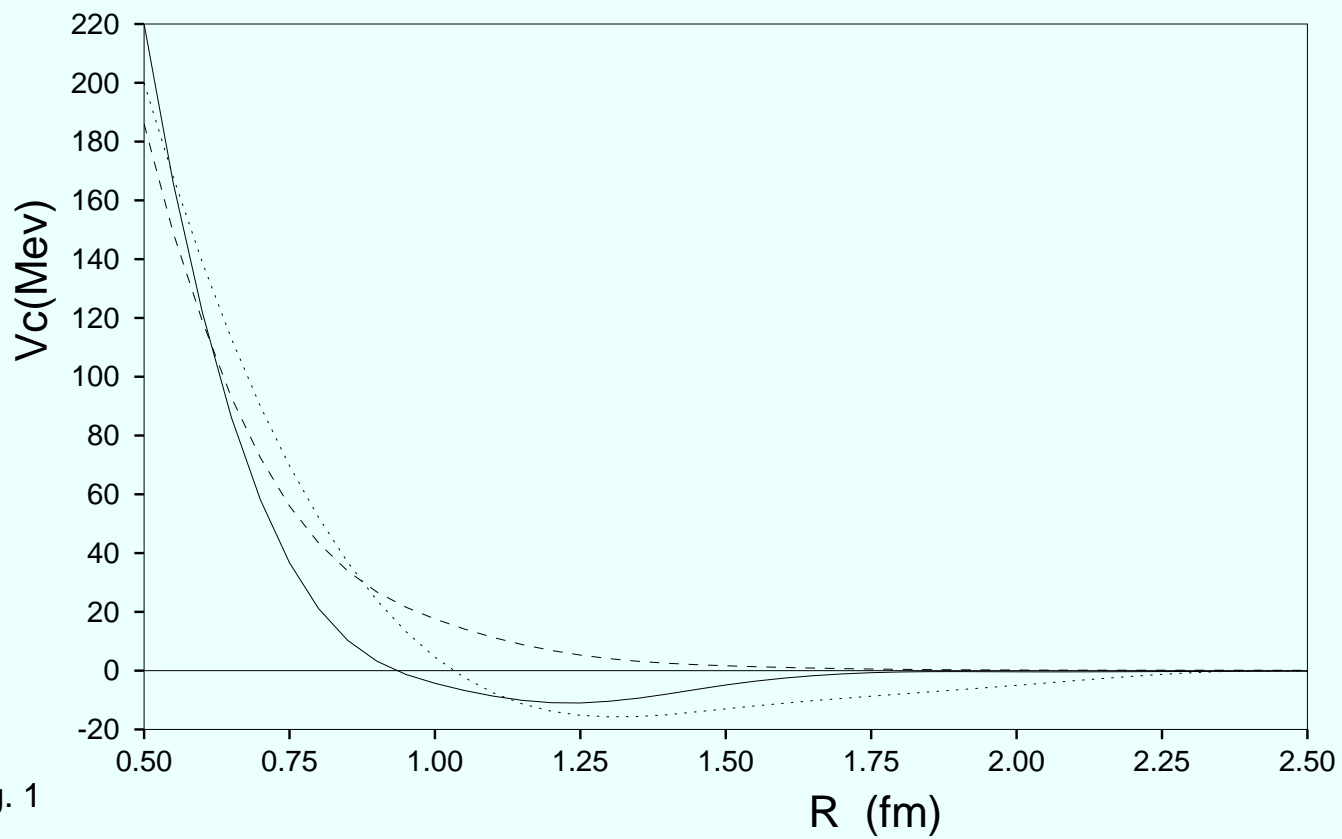


Fig. 1

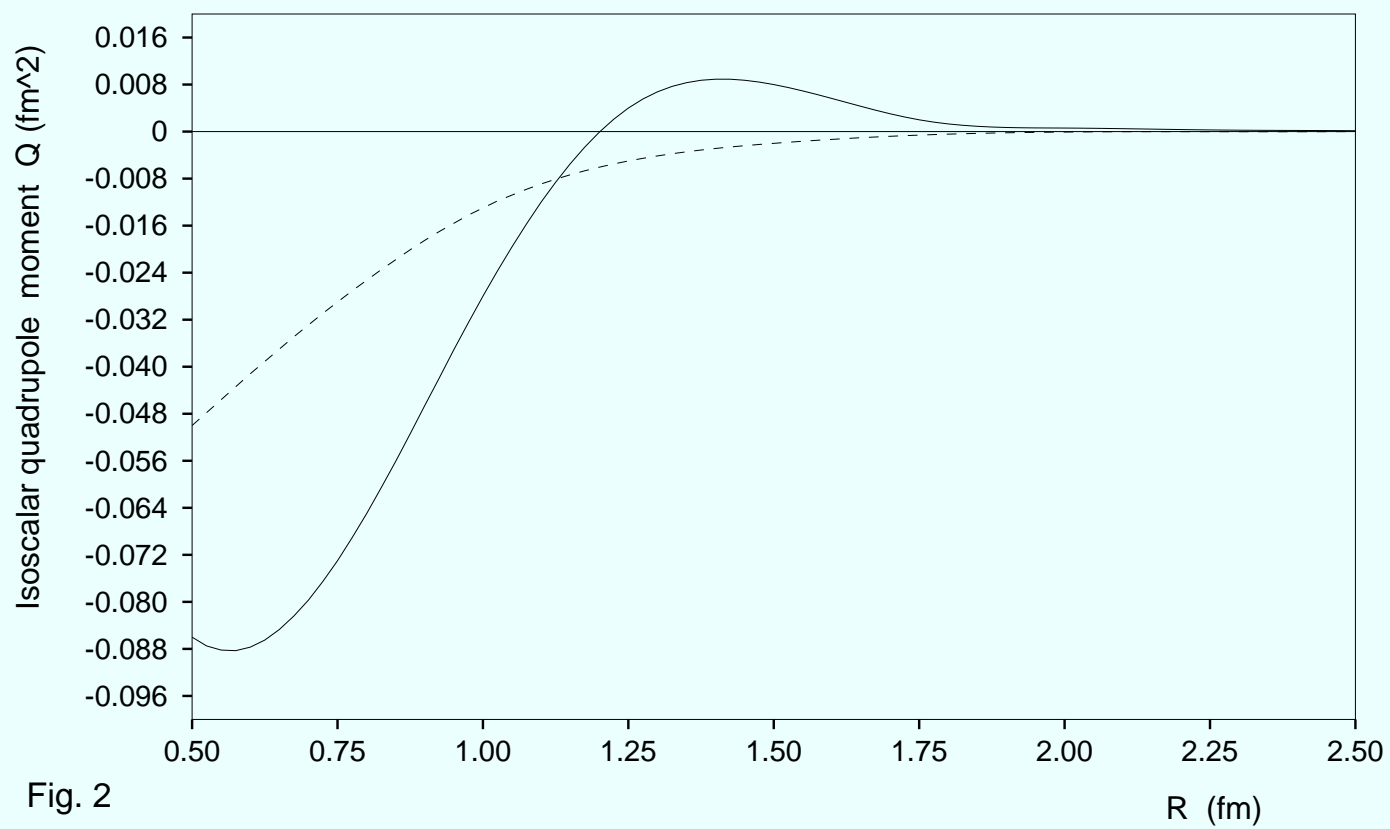


Fig. 2

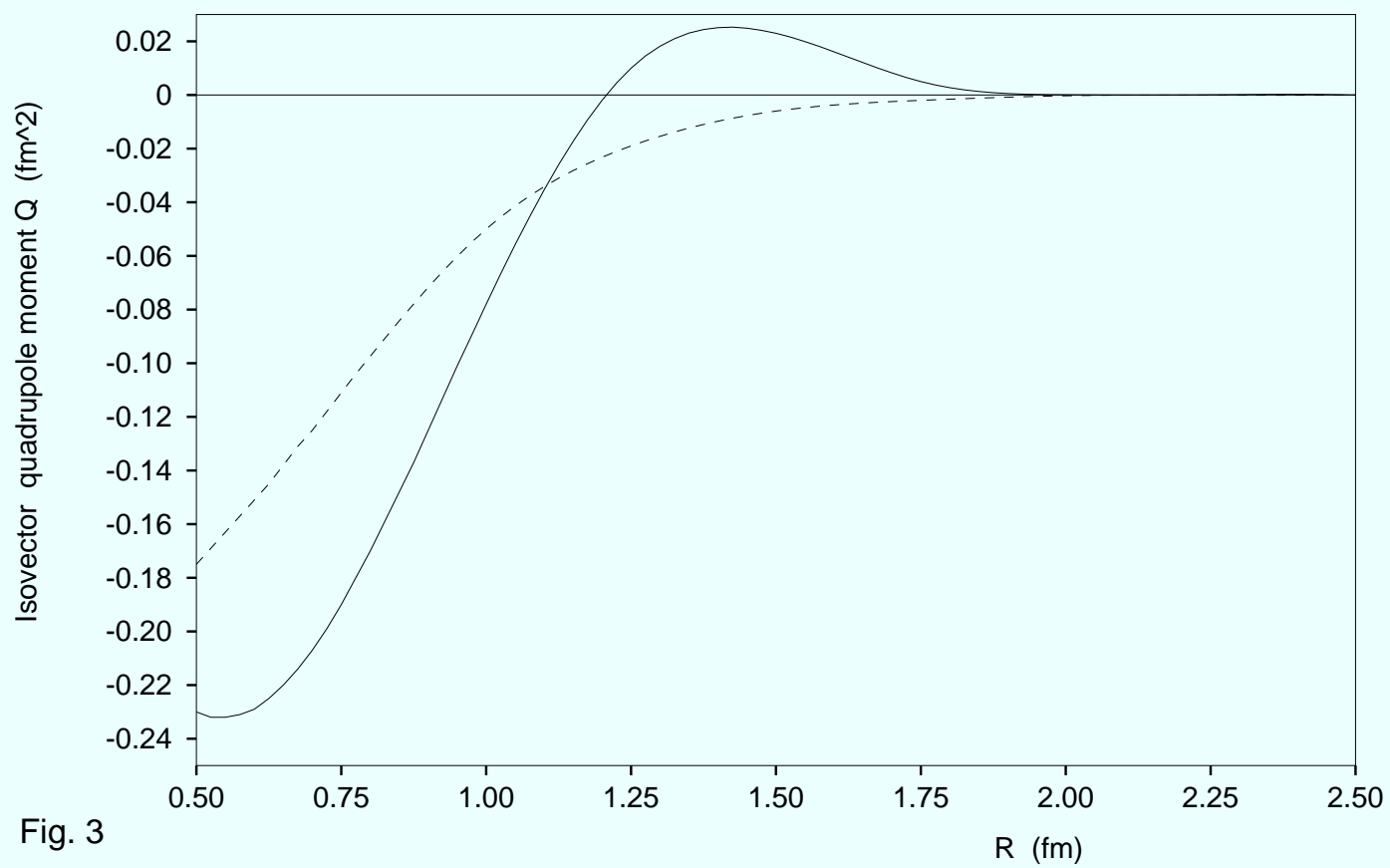


Fig. 3